

## 11.6

# The Form Is “Key”

## Vertex Form of a Quadratic Function

### LEARNING GOALS

In this lesson, you will:

- Determine key characteristics of parabolas using a graphing calculator.
- Determine key characteristics of parabolas given their equations in standard form.
- Determine key characteristics of parabolas given their equations in factored form.
- Determine key characteristics of parabolas given their equations in vertex form.
- Write equations of parabolas given key characteristics of their graphs.

### KEY TERM

- vertex form

Once upon a time, people believed that a god, named Atlas, held up all of the sky, or the “Celestial Sphere.” And it was assumed that a deity made thunder and lightning and caused the Sun to shine and even crops to grow.

Of course, now we know that nothing whatsoever holds up the sky, except for gravity and the laws of physics. We know what makes lightning and thunder, and we know what causes crops to flourish or not. And we even know what causes the Sun to shine—and why it won’t continue to shine forever.

**PROBLEM 1** Analyzing Forms



1. For each function, use a graphing calculator to:

- complete the table of values.
- sketch the graph in the space shown using a window with the given bounds.
- tell whether the parabola opens up or down.
- determine the location of the vertex.
- determine the zero(s).
- determine the y-intercept.

Before you pick up your calculator, think about each given function. What information does it give you?

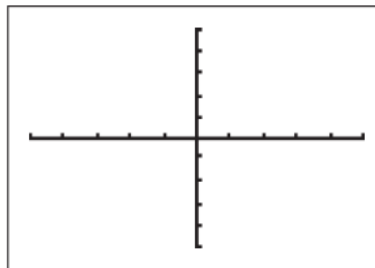


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a.  $f(x) = x^2 + 2x - 3$

| x  | f(x) |
|----|------|
| -2 |      |
| -1 |      |
| 0  |      |
| 1  |      |
| 2  |      |

$[-10, 10] \times [-10, 10]$



parabola opens: \_\_\_\_\_

vertex: \_\_\_\_\_

zero(s): \_\_\_\_\_

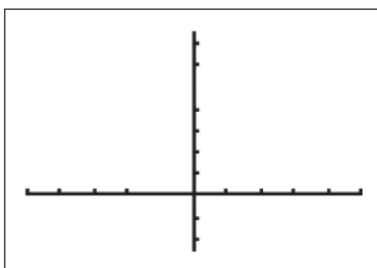
y-intercept: \_\_\_\_\_

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b.  $f(x) = -2x^2 + 6x + 20$

$[-10, 10] \times [-10, 30]$

| $x$ | $f(x)$ |
|-----|--------|
| -2  |        |
| -1  |        |
| 0   |        |
| 1   |        |
| 2   |        |



parabola opens: \_\_\_\_\_

vertex: \_\_\_\_\_

zero(s): \_\_\_\_\_

y-intercept: \_\_\_\_\_

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The quadratic functions in Question 1 are written in standard form,  $f(x) = ax^2 + bx + c$ . You learned about the standard form of a quadratic function earlier in this chapter.



2. In Question 1, you determined whether the parabola opens up or down, the location of the vertex, the zeros, and the y-intercept using a graphing calculator.

Which, if any, of those key characteristics can you determine directly from a quadratic function when it is written in standard form?



3. For each function, use a graphing calculator to:

- complete the table of values.
- sketch the graph in the space shown using a window with the given bounds.
- tell whether the parabola opens up or down.
- determine the location of the vertex.
- determine the zero(s).
- determine the y-intercept.

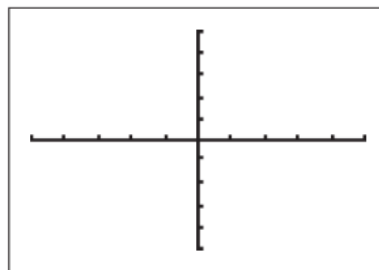
What information does a function in this form tell you?



a.  $f(x) = (x - 1)(x + 3)$

| x  | f(x) |
|----|------|
| -2 |      |
| -1 |      |
| 0  |      |
| 1  |      |
| 2  |      |

$[-10, 10] \times [-10, 10]$



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parabola opens: \_\_\_\_\_

vertex: \_\_\_\_\_

zero(s): \_\_\_\_\_

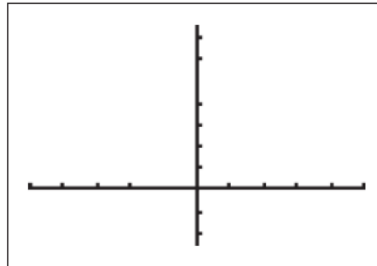
y-intercept: \_\_\_\_\_

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b.  $f(x) = -2(x + 2)(x - 5)$

| $x$ | $f(x)$ |
|-----|--------|
| -2  |        |
| -1  |        |
| 0   |        |
| 1   |        |
| 2   |        |

$[-10, 10] \times [-10, 30]$



parabola opens: \_\_\_\_\_

vertex: \_\_\_\_\_

zero(s): \_\_\_\_\_

y-intercept: \_\_\_\_\_

4. Compare your answers in Question 1 with your answers in Question 3.  
What do you notice?

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The quadratic functions in Question 3 are written in factored form,  $f(x) = a(x - r_1)(x - r_2)$ .  
You learned about the factored form of a quadratic function earlier in this chapter.



5. What key characteristics can you determine directly from a quadratic function when it is written in factored form?



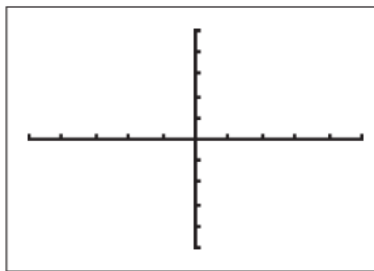
6. For each function, use a graphing calculator to:

- complete the table of values.
- sketch the graph in the space shown using a window with the given bounds.
- tell whether the parabola opens up or down.
- determine the location of the vertex.
- determine the zero(s).
- determine the  $y$ -intercept.

a.  $f(x) = (x + 1)^2 - 4$

| $x$ | $f(x)$ |
|-----|--------|
| -2  |        |
| -1  |        |
| 0   |        |
| 1   |        |
| 2   |        |

$[-10, 10] \times [-10, 10]$



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parabola opens: \_\_\_\_\_

vertex: \_\_\_\_\_

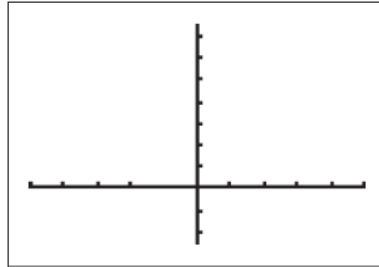
zero(s): \_\_\_\_\_

$y$ -intercept: \_\_\_\_\_

b.  $f(x) = -2(x - 1.5)^2 + 24.5$

$[-10, 10] \times [-10, 30]$

| x  | f(x) |
|----|------|
| -2 |      |
| -1 |      |
| 0  |      |
| 1  |      |
| 2  |      |



parabola opens: \_\_\_\_\_

vertex: \_\_\_\_\_

zero(s): \_\_\_\_\_

y-intercept: \_\_\_\_\_

7. What do you notice when you compare the functions in Question 6 with the functions in Question 1?

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The quadratic functions in Question 6 are written in *vertex form*. A quadratic function written in **vertex form** is in the form  $f(x) = a(x - h)^2 + k$ , where  $a \neq 0$ .

8. What does the variable  $h$  represent in the vertex form of a quadratic function?
9. What does the variable  $k$  represent in the vertex form of a quadratic function?

Look back at the two quadratic functions. Do you see how this form of the function tells you about the vertex?



10. What key characteristics can you determine directly from the quadratic function when it is written in vertex form?

### Talk the Talk



1. Simone, Teresa, Jesse, Leon, and David are working together on the problem shown.

Write a quadratic function to represent a parabola that opens upward and has a vertex at  $(-6, -4)$ .

**Simone**

My function is

$$s(x) = 3(x + 6)^2 - 4.$$

**Teresa**

My function is

$$t(x) = \frac{1}{4}(x + 6)^2 - 4.$$

**Jesse**

My function is

$$j(x) = -3(x + 6)^2 - 4.$$

**David**

My function is

$$d(x) = (x + 6)^2 - 4.$$

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**Leon**

My function is

$$f(x) = 2(x - 6)^2 + 4.$$

- Use a graphing calculator to graph each student's function. What are the similarities among all the graphs? What are the differences among the graphs?
- How is it possible to have more than one correct function?
- What would you tell Jesse and Leon to correct their functions.





- d. How many possible functions can you write for the parabola described in this problem? Explain your reasoning.



2. Use a graphing calculator to rewrite each quadratic function. First, determine the vertex of each and write the function in vertex form. Then, determine the zero(s) of each and write the function in factored form.

a.  $h(x) = x^2 - 8x + 12$

vertex: \_\_\_\_\_

vertex form: \_\_\_\_\_

zero(s): \_\_\_\_\_

factored form: \_\_\_\_\_

b.  $r(x) = -2x^2 + 6x + 20$

vertex: \_\_\_\_\_

vertex form: \_\_\_\_\_

zero(s): \_\_\_\_\_

factored form: \_\_\_\_\_

c.  $w(x) = -x^2 - 4x$

vertex: \_\_\_\_\_

vertex form: \_\_\_\_\_

zero(s): \_\_\_\_\_

factored form: \_\_\_\_\_

d.  $c(x) = 3x^2 - 3$

vertex: \_\_\_\_\_

vertex form: \_\_\_\_\_

zero(s): \_\_\_\_\_

factored form: \_\_\_\_\_

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3. Identify the form(s) of each quadratic function as either standard form, factored form, or vertex form. Then state all you know about each quadratic function's key characteristics, based only on the given equation of the function.

a.  $f(x) = -(x - 1)^2 + 9$

\_\_\_\_\_

b.  $f(x) = x^2 + 4x$

\_\_\_\_\_

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c.  $f(x) = -\frac{1}{2}(x - 3)(x + 2)$

\_\_\_\_\_

d.  $f(x) = x^2 - 5$

\_\_\_\_\_

4. Use the given information to write a possible equation for each quadratic function.

a. The zeros are  $-4$  and  $6$ , and the parabola opens down.

You can use a graphing calculator to check to see if your functions satisfied the given conditions.



b. The vertex is  $(0, 3)$ , and the parabola opens up.

c. The vertex is  $(-1, 1)$ , and the parabola opens down.



d. The zeros are  $0$  and  $2$ , and the parabola opens up.

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5. Complete the graphic organizers on the next two pages using the standard form of the function given. Check the appropriate boxes for each function form.



Be prepared to share your solutions and methods.

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**Standard Form**

Equation:  $f(x) = x^2 + 2x - 3$

Identify the key features from standard form:

- parabola opens up/down
- location of vertex
- zeros
- y-intercept

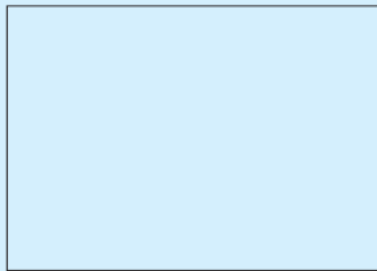
**Factored Form**

Equation: \_\_\_\_\_

Identify the key features from factored form:

- parabola opens up/down
- location of vertex
- zeros
- y-intercept

**Graph of the Quadratic Function**



**Vertex Form**

Equation: \_\_\_\_\_

Identify the key features from vertex form:

- parabola opens up/down
- location of vertex
- zeros
- y-intercept

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**Standard Form**

Equation:  $f(x) = -2x^2 + 6x + 20$

Identify the key features from standard form:

- parabola opens up/down
- location of vertex
- zeros
- y-intercept

**Factored Form**

Equation: \_\_\_\_\_

Identify the key features from factored form:

- parabola opens up/down
- location of vertex
- zeros
- y-intercept

**Graph of the Quadratic Function**

**Vertex Form**

Equation: \_\_\_\_\_

Identify the key features from vertex form:

- parabola opens up/down
- location of vertex
- zeros
- y-intercept